Adaptive Control of the Boost DC-AC Converter

Carolina Albea, Carlos Canudas-de-Wit*, Francisco Gordillo

***Abstract*— In this paper an adaptive control is designed for the nonlinear boost inverter in order to cope with unknown resistive load. This adaptive control is accomplished by using a state observer to one side of the inverter and by measuring the state variables. In order to analyze the stability of the full system singular perturbation analysis is used. The resultant adaptive control is tested by means of simulations.**

I. Introduction

The control of boost DC-AC converters is usually accomplished tracking a reference (sinusoidal) signal. The use of this external signal makes the closed-loop control system to be non-autonomous and thus, making its analysis involved. In [1], [2] a different approach was used: a control law was designed for the boost converter in order to stabilize a limit cycle corresponding to the desired behavior. No external signals were needed. Nevertheless, the use of a boost converter prevents the achievement of zero-crossing signals and, thus, AC current was not achieved. This problem was solved in [3] with the use of a double boost converter as was proposed in [4]. A phase-lock loop was necessary for the correct operation of the circuit as well as for synchronization with the electrical grid. Only the case of known resistive load was considered.

In this paper the previous results are extended to the case of unknown load using an adaptation mechanism. Adaptation mechanism for similar controllers were used in [1] for the case of the buck converter which can be modelled by linear equations. The fact that the boost converter model is nonlinear makes the design of the adaptation law more involved. A state observer for some of the converter variables is designed even when the state variables are measured. In order to analyze the stability of the full system singular perturbation analysis is used. For simplicity, the phase-lock system is not considered in this analysis. The resultant adaptive control is tested by means of simulations.

The rest of the paper is organized as follows: in Sect. II the model of the double boost converter (boost inverter) is presented. Section III states the problem, which is solved in Sect. IV by means of the design of the adaptation mechanism. Section V is devoted to the stability analysis and Sect. VI presents some simulation results. The paper closes with a section of conclusions.

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II. Boost inverter model

The boost inverter is specially interesting because it generates an AC output voltage larger than the its DC input [4]. It is composed of two DC-DC converters and a load connected as shown in Fig. 1. Each converter produces a DC-biased sine wave output, V_a and V_b, so that each source generates an unipolar voltage. The circuit implementation is shown in Fig. 2. Voltages V_a and V_b should present a phase shift equal to 180°, which maximizes the voltage excursion across the load [4].

It is here assumed that:

- all the components are ideal and the currents of the converter are continuous,
- the inductances $L_1 = L_2$, and the capacitances $C_1 = C_2$, are known and symmetric,
- the load R_0 is unknown, and it is needed to be estimated.

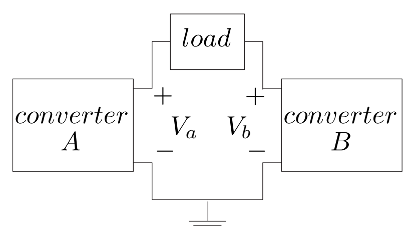

Fig. 1. Basic representation of the boost inverter.

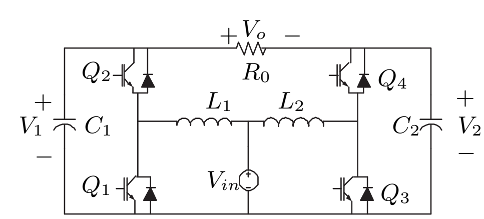

Fig. 2. Ideal Boost DC-AC Converter.

The circuit in Fig. 2 is driven by the transistor ON/OFF inputs Q_i. This yields two modes of operations illustrated in Appendix A. Formally this yields a switched model which is more involved. For control purposes, it is common to use an averaged model described in terms of the mean currents and voltages values. This model is more suited for control because it is described by a "continuous" time smooth and nonlinear ODE. Following [3], this averaging process yields the normalized model described below.

A. Normalized averaged model

Assuming resistive load, a normalized model in terms of the averaged current x_1 and the averaged voltage x_2, for one side of the inverter (see, [3]) is:

$$\begin{aligned} \dot{x}_1 &= -u_1 x_2 + 1 && (1) \\ \dot{x}_2 &= u_1 x_1 - a x_2 + a x_4 && (2) \end{aligned}$$

and the respective normalized model for the other side is obtained similarly by symmetry

$$\begin{aligned} \dot{x}_3 &= -u_2 x_4 + 1 && (3) \\ \dot{x}_4 &= u_2 x_3 + a x_4 - a x_2 && (4) \end{aligned}$$

where here $u_i \in [0,1], i = 1,2$ describes the control inputs. Note that they are also normalized and reflect the mean duty-cycle activation percent of each circuit. They are here treated as "continuous" variables. Parameter $a = \frac{1}{R_0}\sqrt{\frac{L_1}{C_1}}$ depends on the load charge and is assumed to be known.

III. Problem formulation

The control problem is to design a control law for u_1, and u_2, for the system (1)–(2) and (3)–(4) in order to make the output y to oscillate as a sinusoidal signal with a given amplitude i.e.

$$y = x_2 - x_4 \quad \rightarrow \quad y_r = A\cos(\omega t + \varphi)$$

with a pre-specified value for A, and ω. The phase shift φ is no specified.

Under the assumption that a is constant and known, in [3] a nonlinear control law based on Hamiltonian approach was proposed. The design is based on the following change of coordinates:

$$\begin{aligned} \eta_1 &= \frac{x_1^2 + x_2^2}{2} && (5) \\ \eta_2 &= x_1 - a x_2^2 + a x_2 x_4 + \eta_{20} && (6) \\ \eta_3 &= \frac{x_3^2 + x_4^2}{2} && (7) \\ \eta_4 &= x_3 - a x_4^2 + a x_2 x_4 + \eta_{40} && (8) \end{aligned}$$

The controller recalled further below, has as an objective to render the following functions tend to zero

$$\begin{aligned} \Gamma_1 &\equiv \omega^2(\eta_1 - \eta_{10})^2 + (\eta_2 - \eta_{20}^2)^2 - \mu = 0 \\ \Gamma_2 &\equiv \omega^2(\eta_3 - \eta_{30})^2 + (\eta_4 - \eta_{40}^2)^2 - \mu = 0 \end{aligned}$$

orbitally stable. Parameters η_{10}, η_{20} and η_{30}, η_{40} define the respective ellipse centers and ω, μ are related to their size. Based on this definition, the nonlinear control law as proposed in [3] has the following form:

$$u = k(x,a) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_1(x,a) \\ k_2(x,a) \end{bmatrix} \qquad u \in \mathbb{R}^2$$

with $k_1(x,a), k_2(x,a)$ given in Appendix B.

The design is completed with an additional outer loop (PLL) that has the function of achieving a phase shift equal to 180° between the two voltages V_1, and V_2 reaching in that way the desired objective. The goal here is to extent this work to the case of unknown load.

IV. Adaptation law load design

In this section we propose an adaptive law (or a load observer) to cope with load variations and/or uncertainties on the load parameter a. This observer is designed based only on one-side of the circuit, which contains enough information to make this parameter observable. Therefore the use of the full two-side circuit is not necessary at this stage.

The one-side (left) circuit (1)-(2), can be rewritten compactly as:

$$\begin{aligned} \dot{x}_l &= U_l x_l + a B_l y + E_l && (9) \\ y &= x_2 - x_4 && (10) \end{aligned}$$

with $x_l = [x_1, x_2]^T$, and

$$U_l = \begin{bmatrix} 0 & -u_1 \\ u_1 & 0 \end{bmatrix}, B_l = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, E_l = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

In what follows, we assume that both voltages and currents are measurable (either analogically or numerically), and thus accessible for control use.

A. Adaptation law

The proposed adaptation law is composed by: an state observer for one side of the inverter boost, plus an adaptation law for a. It has the following structure:

$$\begin{aligned} \dot{\hat{x}} &= U_l \hat{x} + \hat{a} B_l y + E_l + K(x_l - \hat{x}) && (11) \\ \dot{\hat{a}} &= \beta(x_l, \hat{x}) && (12) \end{aligned}$$

where $K \in \mathbb{R}^{2\times 2}$ is a constant design matrix, and $\beta(x_l, \hat{x})$ is the adaptation law to be designed. Note, that even if x_l is accessible, the adaptation law designed here requires the additional (or extended) state observer. This will become clear during the analysis of the error equation system, as studied below.

B. Error equation

Assume that a is a constant parameter ($\dot{a} = 0$) (or that change slowly $\dot{a} \approx 0$) and define the following error variables:

$$\tilde{x} = x_l - \hat{x}, \quad \tilde{a} = a - \hat{a}, \quad \dot{\tilde{a}} = -\dot{\hat{a}}.$$

Error equation are now derived from (9)–(10) together with (11)–(12)

$$\begin{aligned} \dot{\tilde{x}} &= K\tilde{x} + \tilde{a} B_l y && (13) \\ \dot{\tilde{a}} &= -\beta(x_l, \hat{x}) && (14) \end{aligned}$$

Let K be of the form,

$$K = -\alpha I, \quad \alpha > 0$$

and $P = I$ be the trivial solution of $PK^T + KP = -Q$, with $Q = 2\alpha I$.

Now introducing

$$V = \tilde{x}^T P \tilde{x} + \frac{\tilde{a}^2}{\gamma} \tag{15}$$

it follows that

$$\begin{aligned} \dot{V} &= -\tilde{x}^T Q \tilde{x} + 2\tilde{a}\left(\tilde{x}^T P B_l y + \frac{\dot{\tilde{a}}}{\gamma}\right) \\ &= -\tilde{x}^T Q \tilde{x} + 2\tilde{a}\left(\tilde{x}^T P B_l y - \frac{\dot{\hat{a}}}{\gamma}\right) \end{aligned}$$

The adaptation law is now designed by canceling the terms in square brackets, i.e.

$$\dot{\hat{a}} = \gamma(B_l^T P \tilde{x})y \tag{16}$$

C. Stability properties

The observer and the adaptive law error equations are now fully defined. These equations are:

$$\dot{\tilde{x}} = K\tilde{x} + \tilde{a}B_l y \tag{17}$$

$$\dot{\tilde{a}} = -\gamma(B_l^T P \tilde{x})y \tag{18}$$

The stability properties of these equations follows from the Lyapunov function V defined above. Note that with the choice (16) it follows that

$$\dot{V} = -\tilde{x}^T Q \tilde{x}$$

From standard Lyapunov arguments, it follows that the error variable $\tilde{x}$ and $\tilde{a}$ are bounded. In addition by LaSalle invariant principle, we easily conclude that $\tilde{x} \to 0$, which implies from (18) that $\dot{\tilde{a}} \to 0$.

From (17), and by the property $\tilde{x} \to 0$, and $\dot{\tilde{x}} \to 0$, we have that

$$\tilde{a}B_l y \to 0$$

Note that if y behaves as a sinusoidal as expected from the control problem formulation, the unique asymptotic solution for $\tilde{a}$, is $\tilde{a} = 0$, as $y \not\equiv 0, \forall t \geq 0$.

Note, that in the instants that $y = 0$, (9) does not depends on parameter a.

The following lemma summarizes the results:

Lemma 1: Consider the open-loop system (9)–(10), and assume that its solutions are bounded. The extended observer (11)–(12) has the following properties:

i) The estimated states $\hat{x}$, $\hat{a}$ are bounded.

ii) $\lim_{t\to\infty} \hat{x}(t) = x(t)$.

iii) $\lim_{t\to\infty} \hat{a}(t) = a$, if and only if $y(t) \not\equiv 0, \forall t \geq 0$.

V. Stability of the full closed-loop equations

In the previous section we have presented the stability properties of the extended observer. These properties are independent to the evolution of the system state variables. The stability of the complete system is analyzed in this section.

The open-loop two-sides inverter (1)-(2) and (3)-(4), can be compactly rewritten as:

$$\dot{x} = U(u)x + aBy + E \tag{19}$$

$$y = x_2 - x_4 \tag{20}$$

with $x = [x_1, x_2, x_3, x_4]^T$, and

$$U = \begin{bmatrix} 0 & -u_1 & 0 & 0 \\ u_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -u_2 \\ 0 & 0 & u_2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \end{bmatrix}, E = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

A. Tuned System

The *tuned system* is defined as the ideal *closed-loop* system under the action of the *tuned feedback law* $u^* = k(x, a)$, computed with the *exact* value of a.

The tuned systems given in [3] writes

$$\dot{x} = U(u^*)x + aBy + E \tag{21}$$

$$= U(k(x,a))x + aBy + E \tag{22}$$

$$= f(x) \tag{23}$$

and it achieves an asymptotically orbitally stable periodic solutions, i.e.

$$x^*(t) = x^*(t+T)$$

In [3] it has been shown that the funtions Γ_1 and Γ_2 defined in (9)–(9) tend to zero. They correspond to periodic sinusoidal solutions with period $T = 2\pi/\omega$. Consequently, $y^* = x_2^* - x_4^*$ is also sinusoidal.

B. Closed-loop system

In practice, the control law which is effectively applied depends on the estimation of parameter a. We denote this control law as $\hat{u} = k(x, \hat{a})$. Note that this control law depends on the state x and not on its estimation $\hat{x}$, because the state x is directly measured. The role of $\hat{x}$ is then just to make possible the design of the adaptation law for a.

The closed-loop equation resulting from the use of $\hat{u} = k(x, \hat{a})$ writes, as

$$\dot{x} = U(\hat{u})x + aBy + E \pm U(u^*)x \tag{24}$$

$$= f(x) + [U(\hat{u}) - U(u^*)]\, x \tag{25}$$

$$= f(x) - U(\tilde{u})x \tag{26}$$

where $\tilde{u} = u^* - \hat{u}$. Note that $U(\tilde{u}) = U(\tilde{x}, \tilde{a})$, can be partitioned as follows:

$$U(\tilde{u}) = U(\tilde{x}, \tilde{a}) = \begin{pmatrix} \mathbb{I}\varphi_1(x, a_m)\tilde{a} & 0 \\ 0 & \mathbb{I}\varphi_2(x, a_m)\tilde{a} \end{pmatrix}$$

where $\varphi_i(x, a_m) = \partial_a k_i(x, \tilde{a})|_{\tilde{a}=a_m}, \forall i = 1, 2$. This expression results from the application of the mean-value theorem, with $a_m \in [a_{min}, a_{max}]$, being a value of a in the allowed physical interval. The screw-symmetric matrix $\mathbb{S} = -\mathbb{S}^T$ is defined as

$$\mathbb{S} = diag\{\mathbb{I}, \mathbb{I}\}, \qquad \mathbb{I} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

The term $U(\tilde{u}) = U(x, \tilde{a})$ captures the mismatch between the estimated and the true value of the load. In view of the discussion above, this term has the following property:

Property 1: Let $\mathbb{M} = \{(x, \tilde{a}) : ||x - x^*|| < \epsilon_x, |\tilde{a}| < \epsilon_a\}$, be a compact domain including the asymptotic periodic solutions of the tuned system and the exact load. Then, the function $U(\tilde{u}) = U(x, \tilde{a})$ has $\forall(x, \tilde{a}) \in \mathbb{M}$, the following properties:

i) it is continuous, analytic, and free of singularities
ii) it has the following limits:

$$\lim_{\tilde{u}\to 0} U(\tilde{u}) = \lim_{\tilde{a}\to 0} U(x, \tilde{a}) = 0.$$

Putting together (26) with the observer error system give the complete set of closed-loop equation, with $y = y(x)$

$$\dot{x} = f(x) - U(x, \tilde{a})x \tag{27}$$
$$\dot{\tilde{x}} = -\alpha\tilde{x} + \tilde{a}By \tag{28}$$
$$\dot{\tilde{a}} = -\gamma(B^T P\tilde{x})y \tag{29}$$

where we have substituted $K = -\alpha I$. The stability consideration discussed here will be based on the time-scale separation. The main idea is that with the suited choice of gains (as discussed latter) the observer equation (28)-(29) can be seen as the fast variables and the equation (27) as the slow subsystem. Note again, that this time-scale separation should be enforced by a particular choice of the observer and adaption gains: K and γ.

C. Singular perturbed form

To put the system above in the standard singular perturbation form, we follow the next steps:

- introduce $\bar{a} = \frac{\tilde{a}}{\alpha}$,
- select $\gamma = \alpha^2$
- define $\varepsilon = \frac{1}{\alpha}$

With these considerations, we achieve,

$$\begin{aligned} \dot{x} &= f(x) - U(x, \bar{a})x \\ \varepsilon\dot{\tilde{x}} &= -\tilde{x} + \bar{a}By, \\ \varepsilon\dot{\bar{a}} &= -(B^T P\tilde{x})y. \end{aligned}$$

where $\varepsilon > 0$ being the small parameter. Note that this particular selection of gains imposes relative gains for the adaptation γ, and defines precisely how the observer gain are related to γ. The target system for the slow variables, defined after the change of coordinates (5)–(8) [3], is

$$\begin{aligned} \dot{\eta}_1 &= \omega\eta_2 \\ \dot{\eta}_2 &= -\omega\eta_1 - kx_2\Gamma. \end{aligned}$$

Dividing this equations by ω they achieve a similar form to fast variables equations. As we want that variable x is much slower than z we have to impose

$$\varepsilon \ll \frac{1}{\omega}, \quad \varepsilon \ll \frac{1}{k}.$$

This means that the adaptation gain γ as well as the tuning parameter k should be related to the desired frequency as:

$$\gamma \gg \omega^2, \quad \gamma \gg k^2$$

Letting $z = [\tilde{x}, \bar{a}]^T$ gives the general form

$$\dot{x} = f(x) - U(\tilde{u})x \tag{30}$$
$$\varepsilon\dot{z} = g(x, z) \tag{31}$$

with, $x(t_0) = x^0, x \in \mathbb{R}^3$, $z(t_0) = z^0, z \in \mathbb{R}^5$, and

$$g(z, x) = \begin{bmatrix} -\tilde{x} + \bar{a}By \\ -(B^T P\tilde{x})y \end{bmatrix}$$

According to the singular perturbation analysis, we need to follow the next steps:

1) Find a stationary solution of the *fast* subsystem (31) by finding roots of the equation $g(x, z) = 0$, i.e. $z = \phi(x)$
2) Substitute this solution in the *slow* subsystem (30), and find a the resulting slow system

$$\dot{x} = f(x) - U(\tilde{u}(x, \phi(x)))x$$

3) Check the boundary layer properties of the fast subsystem along one particular solution of $\dot{x} = f(x) - U(\tilde{u}(x, \phi(x)))x$.

D. Slow sub-system

Proceeding to the steps 1 and 2 above requires to find the solution for the algebraic equation

$$0 = g(x, z) = \begin{bmatrix} -\tilde{x} + \bar{a}By \\ -(B^T P\tilde{x})y \end{bmatrix}$$

whose roots are calculated from

$$\begin{aligned} \tilde{x} &= \bar{a}By \\ 0 &= -\bar{a}B^T PBy^2 \end{aligned}$$

note that $B^T PB = 1$, and that the above equation has multiple solutions, i.e

$$\begin{aligned} \tilde{x} &= 0 \\ \bar{a}y^2(x) &= 0 \end{aligned}$$

which means that if $y \equiv 0$, there one solution for $\tilde{x} = 0$, and infinite solutions for $\bar{a}$. However, if $y \not\equiv 0$, for instance the particular *tuned* solution $y^* = A\cos(\omega t)$, then

$$z = \phi(x) = \begin{bmatrix} \tilde{x} \\ \bar{a} \end{bmatrix} = 0$$

become an isolated root. Then for this particular solution, and noticing that $\bar{a} = \frac{a-\hat{a}}{\alpha} = 0$, e.i. $\hat{a} = a$ the slow model writes as:

$$\dot{x} = f(x) - U(x, 0)x = f(x), \tag{32}$$

which is nothing else than the tuned system whose solutions $x(t) = x^*(t)$ are sinusoidal.

E. Boundary layer fast subsystem

Now, the next step is to evaluate the stability of the boundary layer system in the finite time interval $t \in [t_0, t_1]$. This is obtained by evaluating the fast subsystem (31) along one particular solution of the quasi-steady-state $x^*(t), \forall t \in [t_0, t_1]$ (tuned system solutions), and by re-scaling time t to $\tau = (t - t_0)/\varepsilon$.

As a particular solution we consider $y^* = x_2^* - x_4^* = A\cos(\omega t + \varphi)$, which expressed in the stretched time coordinates $\tau = (t - t_0)/\varepsilon$ is:

$$y^* = y^*(\tau, \omega, \varepsilon, t_0) = A\cos(\omega(\varepsilon\tau + t_0) + \varphi)$$

the fast subsystem (31) evaluated along such solution is,

$$\begin{aligned} \frac{d}{d\tau}\hat{\hat{x}}_1 &= -\hat{\hat{x}}_1 \\ \frac{d}{d\tau}\hat{\hat{x}}_2 &= -\hat{\hat{x}}_2 - \hat{\hat{a}}y^* \\ \frac{d}{d\tau}\hat{\hat{a}} &= \hat{\hat{x}}_2 y^* \end{aligned}$$

which can be rewritten as:

$$\frac{d}{d\tau}\hat{z} = J(y^*)\hat{z} = J(\tau, \omega, \varepsilon)\hat{z} \tag{33}$$

with

$$J = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & -y^* \\ 0 & y^* & 0 \end{pmatrix}$$

Under this conditions, system (33) is reduced to the autonomous linear system

$$\frac{d}{d\tau}\hat{z} = J(\tau, \omega, 0)\hat{z} = J(y_0^*)\hat{z} \tag{34}$$

Consider the $y_0^* \in D_x$, with $D_x \triangleq \{x : |y| = |x_2 - x_4| > \alpha_0 > 0\}$, the above system has the following properties.

Property 2: The eigenvalues of $J(y^*)$, for $[t, x^*, z] \in [t_0, t_1] \times D_x \times \mathbb{R}^3$, are all strictly negative, i.e.

$$\lambda_1 = -1 \tag{35}$$

$$\lambda_2 = \mathfrak{Re}\left\{\frac{-1 + \sqrt{1 - 4y^{*2}}}{2}\right\} < 0 \tag{36}$$

$$\lambda_3 = \mathfrak{Re}\left\{\frac{-1 - \sqrt{1 - 4y^{*2}}}{2}\right\} < 0 \tag{37}$$

where $c_1 > 0$ is a constant.

Therefore $J(y^*)$ is Hurwitz in the considered domain. As a consequence, there exists a matrix $P(y^*) = P(y^*)^T > 0$ and a $Q(y^*) > 0$ such that the standard Lyapunov equation holds:

$$P(y^*)J(y^*) + J(y^*)^T P(y^*) = -Q(y^*)$$

From standard Lyapunov arguments, it follows that for all $t \in [t_0, t_1]$

$$||\hat{z}(t, \varepsilon)|| \leq c_1 \exp\left\{-\lambda_{min}(Q(y^*))\left(\frac{t - t_0}{\varepsilon}\right)\right\}$$

Tikhonov's theorem, see [5], can now be advocated to summarize the previous result.

Theorem 1: There exists a positive constant ε^* such that for all $y_0^* \in D_x$, and $0 < \varepsilon < \varepsilon^*$, the singular perturbation problem of (30)-(31) has a unique solution $x(t, \varepsilon)$, $z(t, \varepsilon)$ on $[t_0, t_1]$, and

$$x(t, \varepsilon) - x^*(t) = O(\varepsilon) \tag{38}$$

$$z(t, \varepsilon) - \hat{z}^*(t/\varepsilon) = O(\varepsilon) \tag{39}$$

hold uniformly for $t \in [t_0, t_1]$, where $\hat{z}^*(\tau)$ is the solution of the boundary layer model (34). Moreover, given any $t_b > t_0$, there is $\varepsilon^{**} \leq \varepsilon^*$ such that

$$z(t, \varepsilon) = O(\varepsilon)$$

holds uniformly for $t \in [t_b, t_1]$ whenever, $\varepsilon < \varepsilon^{**}$.
Extension of this result to infinite time interval, requires prove that the boundary layer system is exponential stable in a neighborhood of the tuned slow solution $x^*(t)$ for all $t \geq t_0$. This may not be a trivial demonstration, and it will be left for further investigation. Instead, we demonstrated using simulation the effectiveness of this approach.

VI. Simulations

The following simulations are made considering $Vin = 10V$, $R_0 = 100\Omega$, $L_1 = L_2 = 100\mu H$, $C_1 = C_2 = 100\mu F$. The desired output of the circuit is $V_{out} = 40 \sin 50t \ V$.

In order to obtain this voltage, the parameters are $a = 0.001$, $\omega = 0.0314$, $A = 2$, $k = 1.2$ and $\eta_{20} = \eta_{40} = 0$. The ellipse parameters result according to [3] are $\eta_{10} = \eta_{30} = 51.14$, $\mu = 0.395$. The estimated value of parameter a will be $\widehat{a} = 0.0077$ ($R_0 = 130\Omega$), i.e., a 23% error.

Fig. 3 shows the output voltage evolution. Note that the circuit achieves the desired behavior. The time scale is the real time scale, without variable change.

The adaptation of the parameter a is represented in the Fig. 4, moreover, it tests the equation (39). The convergence of slow state variable $x_2(t, \varepsilon) - x_2^*(t)$ to $O(\varepsilon)$ with different ε is shown in the Fig. 5, (equation (38)). Notice the scale value for the vertical axis.

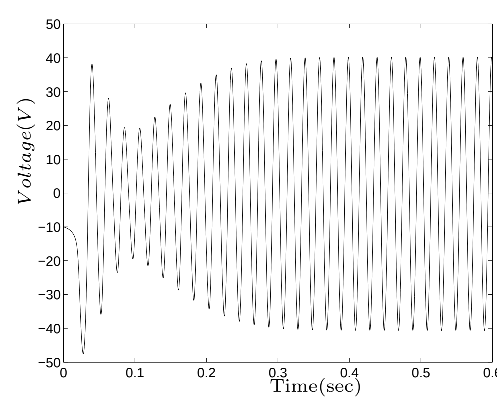

Fig. 3. Output voltage with adaptation of a perturbation of a 23%

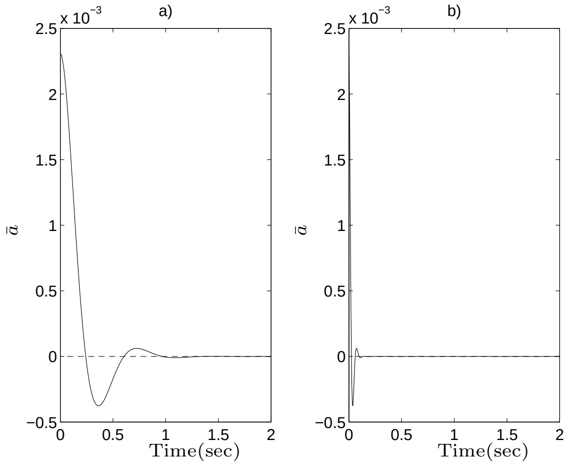

Fig. 4. Time-evolution of the fast variable $\bar{a}$ (solid) and $\bar{a}^*$ (dashed) with $\varepsilon = 0.1$ in *a*) and $\varepsilon = 0.01$ in *b*)

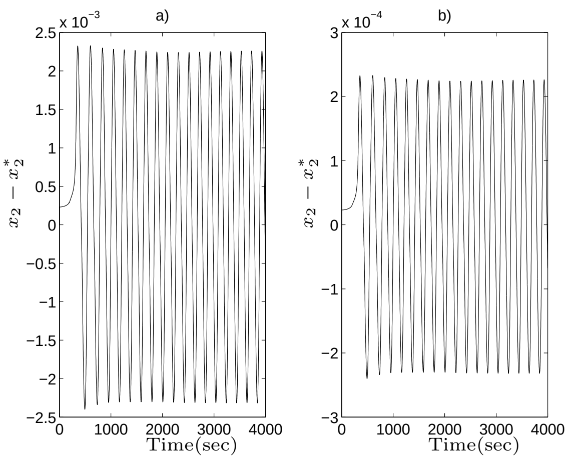

Fig. 5. Convergence of $x_2 - x_2^*$ with $\varepsilon = 0.1$ in *a*) and $\varepsilon = 0.01$ in *b*)

VII. Conclusions

An adaptive control for unknown load is presented for a nonlinear boost inverter. The method is based on using a state observer to one side of the inverter and by knowing that the state variables are measured. The stability of the complete system is proved putting the system in the standard singular perturbation form, hence we obtained a relationship between the adaptation gain, γ, the observer matrix parameter, α, and the perturbed variable parameter, ε. Another important relationship between the perturbed variable parameter, ε, and the system frequency, ω, was achieved in the analysis of the boundary layer fast subsystem. Finally, the stability is established by means of Tikhonov's theorem.

Open problem is the extension of this result to infinite time interval.

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Appendix

A. Operation modes

The two operation modes of the boost inverter are shown in Fig. 6.

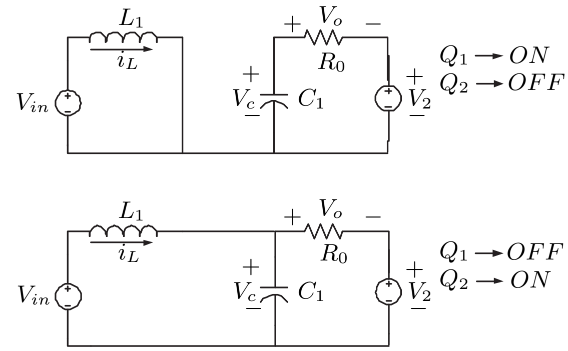

Fig. 6. Operation status.

The equations of the system are

$$L_1 \frac{di_{L_1}}{dt} = -u_1 v_{C_1} + V_{in} \tag{40}$$

$$C_1 \frac{dv_{C_1}}{dt} = u_1 i_{L_1} - \frac{v_{C_1}}{R} + \frac{v_{C_2}}{R_0} \tag{41}$$

where u_1 is a continuous variable that control the transistor states.

In order to simplify the study, the system (40)–(41) is normalized as (1)–(2) by using the change of variables: $x_1 = \frac{1}{V_{in}}\sqrt{\frac{L_1}{C_1}} i_{L_1}$, $x_2 = \frac{v_{C_1}}{V_{in}}$ and defining the new time variable with $\tilde{t} = \frac{1}{\sqrt{L_1 C_1}} t$ and $a = \frac{1}{R_0}\sqrt{\frac{L_1}{C_1}}$.

B. Control law proposed in [3]

$$\begin{aligned}
k_1(x,a) &= \frac{1 + 2a^2x_2^2 - 3a^2x_2x_4 + a^2x_4^2 + a^2x_2\dot{x}_4}{x_2 + 2ax_1x_2 - ax_4x_1} \\
&\quad + \frac{k\Gamma_1(\eta_2 - \eta_{20}) + \omega^2(\eta_1 - \eta_{10})}{x_2 + 2ax_1x_2 - ax_4x_1} \\
k_2(x,a) &= \frac{1 + 2a^2x_4^2 - 3a^2x_2x_4 + a^2x_2^2 - a^2x_4\dot{x}_2}{x_4 - 2ax_3x_4 + ax_2x_3} \\
&\quad + \frac{-k\Gamma_2(\eta_4 - \eta_{40}) + \omega^2(\eta_3 - \eta_{30})}{x_4 - 2ax_3x_4 + ax_2x_3}
\end{aligned}$$

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